

Exercise 5

Verify property (9) of moduli in Sec. 5.

Solution

$$\begin{aligned}\left|\frac{z_1}{z_2}\right|^2 &= \left(\frac{z_1}{z_2}\right) \overline{\left(\frac{z_1}{z_2}\right)} \\ &= \left(\frac{z_1}{z_2}\right) \left(\frac{\bar{z}_1}{\bar{z}_2}\right) \\ &= \frac{z_1 \bar{z}_1}{z_2 \bar{z}_2} \\ &= \frac{|z_1|^2}{|z_2|^2} \\ &= \left(\frac{|z_1|}{|z_2|}\right)^2\end{aligned}$$

Therefore,

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \tag{9}$$

for any complex numbers, z_1 and z_2 .